

Potential evaporation and evapotranspiration data provided by SILO

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1 Introduction

SILO is a database of historical climate data for Australia. Observational data collected by the Bureau of Meteorology are processed to create daily datasets which are ready to use. Mathematical algorithms are used to estimate missing data and construct gridded datasets, so the derived products are both spatially and temporally complete.

SILO provides temporal (point) and spatial datasets for rainfall, maximum and minimum temperatures, evaporation, evapotranspiration, solar radiation, mean sea level pressure, vapour pressure and relative humidity. This report details the evaporation and evapotranspiration estimates provided by SILO.

Evaporation and evapotranspiration are affected by a wide variety of environmental and climatic effects. Users are urged to carefully consider the attributes of each evaporation (or evapotranspiration) variable to ensure the data chosen are fit for purpose. The aim of this report is to assist users in their choice of variable and introduce them to the range of factors that may need consideration. For example, users should note the following:

- Gridded estimates of Class A pan evaporation for years after 2010 are unreliable due to the declining density of the pan observation network.
- We recommend the synthetic pan estimate (Pan_{syn}) be used in water balance and plant growth models with weekly to monthly time steps. The synthetic estimate should generally be usable over these timescales. The *FAO56* estimate may also be a viable alternative for such applications.
- The target application's time scale may influence the choice of variable and data quality requirements. For example, if a model utilising evapotranspiration data is expected to approach the truth at a daily time scale then it may be necessary to account for factors which can influence evapotranspiration on a daily or even sub-daily basis, for example wind speed.
- If a model was calibrated using a given measure of evaporation or evapotranspiration and the measure used is subsequently changed to another, it may be necessary to recalibrate the model parameters to compensate for changing the input data.

1.1 Evaporation

Evaporation is a collective term covering all processes in which liquid water is transferred as water vapour to the atmosphere (McMahon *et al.*, 2013). In the current context it refers to water vaporisation from water or bare-soil surfaces, including vaporised water intercepted by vegetation.

The rate of actual evaporation is determined by:

- the net available heat
- the wetness of the evaporating surface
- the physical state of the surrounding air, as determined by its temperature, vapour pressure and its velocity, (Monteith, 1991).

Evaporation occurs when, in addition to the energy needed for the latent heat of vaporisation, there is a process to remove the water vapour from the evaporating surface. Radiant energy and turbulent transfer of water vapour away from the evaporating surface

are the two processes mainly responsible for enabling the evaporation process. The main resistance to the evaporation flux is a thin non-turbulent layer of air, about 1 to 3 mm thick, next to the surface. This is known as aerodynamic or atmospheric resistance.

Potential evaporation represents an upper limit to evaporation from a moist surface, defined by the atmospheric conditions and saturation vapour pressure at the actual surface temperature (Granger, 1989).

1.2 Evaporation variables provided by SILO

SILO provides the following estimates of evaporation:

- Class A pan observations (Pan_{obs}):
Potential evaporation is measured by the amount of water vaporised from a standard Class A pan over 24 hours – normally 9:00am to 9:00am the following day. Data reported on the measurement day is assigned to the day prior when most of the evaporation is likely to have occurred.
- Synthetic estimate of Class A pan evaporation (Pan_{syn}):
Rayner *et al.* (2005) developed a regression method for estimating Class A pan evaporation from gridded datasets of solar radiation, minimum and maximum temperatures, and vapour pressure. The model is spatially and seasonally-varying, with regression coefficients computed independently for each pixel and month, thereby implicitly accounting for some season and spatial variability due to wind.
- Morton's shallow lake evaporation (M_{lake}):
Morton (1983a) developed a model for evaporation over large, shallow lakes (less than 30m deep and more than 300m wide). Changes in seasonal subsurface heat storage in the water body are neglected and the model is only applicable to lakes with large surface areas as advection energy is not accounted for. The minimum lake size is based upon the typical reach of boundary effects (Morton, 1983a), however Chiew *et al.* (2003) recommended a minimum lake size of 1 km.

2 Evapotranspiration

Transpiration represents water vaporisation from plants, specifically from within leaves of a plant via water vapour flux through leaf stomata (Dingman, 1992). The term evapotranspiration (ET) is the combination of evaporation and transpiration (Allen *et al.*, 1998).

The rate of evapotranspiration is determined by a complex combination of plant physiology and environmental conditions. For example, the degree of stomatal opening in the leaves regulates transpiration (McMahon *et al.*, 2013) and consequently affects the evapotranspiration flux. To eliminate the impact of plant specific characteristics on the evapotranspiration estimate, "potential" or "reference" values are typically used:

- potential evapotranspiration (ET_p) represents the evapotranspiration rate of a short green crop, completely shading the ground, of uniform height and with adequate

water status in the soil profile (Penman, 1948). Note that in the definition of potential evapotranspiration, the rate is not related to a specific crop.

Potential evapotranspiration represents evapotranspiration that would occur if soil water were unlimited and is therefore limited by the amount of solar radiation available at the surface. In contrast, actual evapotranspiration is often limited by availability of water in the soil.

- reference evapotranspiration (ET_0) is the evapotranspiration from a hypothetical reference crop with an assumed crop height of 0.12 m, a fixed surface resistance of 70 sec/m and an albedo of 0.23, closely resembling the evapotranspiration from an extensive surface of green grass of uniform height, actively growing, well-watered, and completely shading the ground (Allen *et al.*, 1998, Irmak and Haman, 2003). Grass is defined as the reference crop and it is assumed to be free of water stress and disease.

Practically, the estimation of the evapotranspiration rate for a specific crop requires first calculating potential or reference evapotranspiration and then applying the appropriate crop coefficients (K_c) and soil moisture status to estimate actual crop evapotranspiration (ET_a) with this typically being calculated within process models of plant growth.

2.1 Potential evapotranspiration variables provided by SILO

SILO provides the following estimates of potential evapotranspiration (ETp) :

- Penman-Monteith (short crop) reference evapotranspiration (*FAO56*): Evapotranspiration over a reference grass surface with height = 0.12m, albedo = 0.23, and a fixed plant-and-surface resistance (Allen *et al.*, 1998). The surface area is assumed to be large enough so that advection energy effects are negligible.

This estimate uses the Penman-Monteith equation and is called *FAO56* because it is documented in Food and Agriculture Organization Paper No. 56 (Allen *et al.*, 1998).

The *FAO56* estimate is mainly used for irrigation purposes: multiplying the reference value by a crop coefficient will yield an estimate of the actual crop evapotranspiration for an extensive irrigated field of the given crop.

- Penman-Monteith (tall crop) reference evapotranspiration (*ASCE*): This is the “tall crop” version of *FAO56*; it assumes the crop height is 0.5m (ASCE, 2000).
- Morton’s point potential evapotranspiration (M_{pp}): Potential evapotranspiration over a wet surface that is so small the local evapotranspiration would not affect the surrounding environment. The model assumes latent and sensible heat transfer near the surface only occurs via convection (Wang *et al.*, 2001).

Chiew and Leahy (2003) found that M_{pp} is similar to Class A pan evaporation.

- Morton’s wet environment areal potential evapotranspiration (M_{wet}):

Potential evapotranspiration over a large area, assuming an unlimited supply of water. The model assumes upwind effects are negligible and local variations are ignored, so the estimate is an areal average (Wang *et al.*, 2001).

Chiew and McMahon (1991) found M_{wet} is similar to *FAO56* in a wet climate but lower than *FAO56* in a dry climate. Chiew and Leahy (2003) found that M_{wet} is similar to *FAO56* in the coastal areas of south-eastern and eastern Australia.

- Morton's areal actual evapotranspiration (M_{aa}):
Actual evapotranspiration over a large area, given the prevailing soil-water conditions. The model assumes upwind effects are negligible and local variations are ignored, so the estimate is an areal average (Wang *et al.*, 2001).

The formulae used in calculating the *FAO56*, *ASCE* and various Morton's estimates are provided in Appendices A and B. The source code for computing these estimates is provided in Appendix C.

3 A guide for choosing which variable to use

Selecting the most appropriate estimate of potential evaporation or potential evapotranspiration for a given application requires careful consideration of factors such as data quality and assumptions underlying the various models.

Observational data from Class A pans are attractive in that they are real-world observations which inherently include all local effects such as solar radiation, temperature, wind and vapour pressure deficit. However the following limitations should be noted:

- the data represent potential evaporation given an unlimited supply of water
- conversion factors can be used to estimate evapotranspiration over land or evaporation over water, given the observational pan evaporation. However the conversion factors are not constant, varying with pan type and local conditions such as the setting of the pan and its surrounding environment (Allen *et al.*, 1998). In other words pan data may not be representative of the actual evaporation or evapotranspiration in the surrounding area.
- pan observations exhibit the same types of errors which affect most observational climate data (such as transcription errors, random observer error, systematic instrument bias *etc.*), however they are also subject to errors specific to pan observations. Examples include: (i) un-flagged accumulations occur in the data, especially at those locations where the pan is not measured on weekends; (ii) quantised measurements (*i.e.* values occurring at discrete intervals) can occur if the pan is not measured or refilled with due care (Robinson, 1998); and (iii) inaccurate measurements can arise if pan operators do not correctly adjust for rainfall (or allow the pan to overflow).
- long term records are not available. While a limited number of stations recorded pan evaporation before 1970, bird guards were not installed until the early 1970's. SILO does not provide observational data from a given site for dates prior to the installation of a bird guard as the data may not be reliable (the Australian sunken tank measurements are probably unreliable given the possibility of undetected leaks).
- bird guards can reduce the measured potential by 7% from the physically possible (van Dijk, 1985). The pan body itself can also impact the measurement.

- the number of stations recording pan data has declined in recent years (Figure 1). As the accuracy of interpolated estimates is critically dependent on the density of the observational network, the declining station density is significantly reducing the accuracy of SILO's Class A pan evaporation gridded datasets. Furthermore, the evolving configuration of recording stations complicates any sort of time series analysis of the data. Clients should consider using SILO's synthetic estimate (Pan_{syn}) as an alternative.
- the impact of declining station numbers in recent years is further exacerbated by the delay in obtaining Class A pan observations from the source. While SILO obtains a daily feed of observational data from the Australian Bureau of Meteorology, not all pan observations are available in near real time. There can be a delay of up to 6 months before the data from some stations are available electronically.

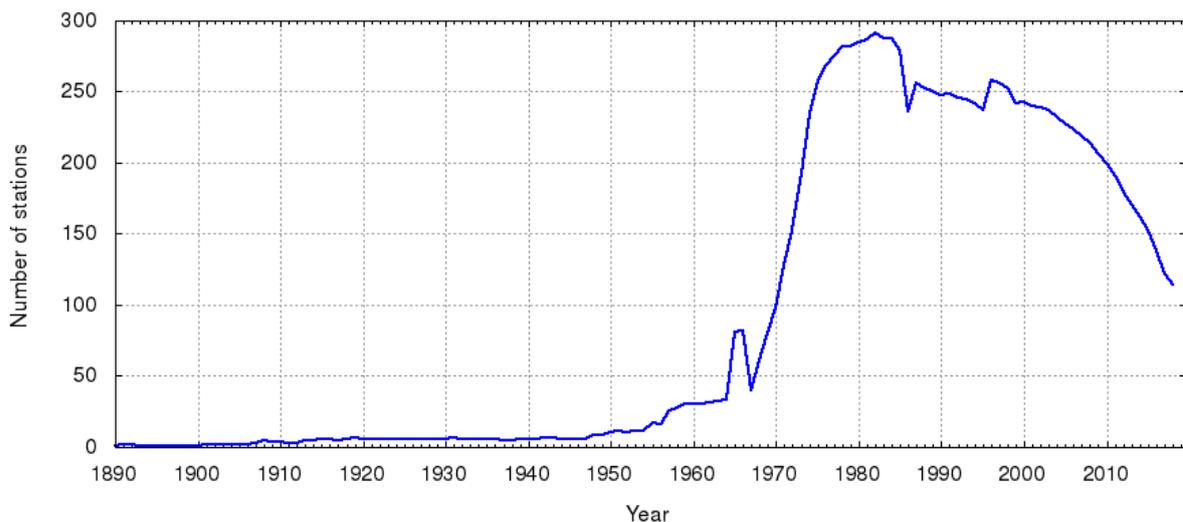


Figure 1. Number of stations within Australia (and surrounding islands) reporting Class A pan evaporation.

The synthetic estimate (Pan_{syn}) is computed from a regression of daily pan data (for each month) with vapour pressure deficit and solar radiation. The vapour pressure deficit is calculated using daily 9am vapour pressure (VP), and the saturation vapour pressure computed using minimum and maximum temperatures, weighted 0.75 towards maximum temperature. Regression parameters were fitted at point locations and interpolated to enable evaluation of Pan_{syn} at any grid cell.

The Pan_{syn} gridded rasters are computed on a pixel-by-pixel basis, using interpolated values of all input variables and gridded parameters. The number of data values used to construct gridded rasters for the four input variables is shown in Figure 2. We recommend clients consider using the synthetic estimate because the Pan_{syn} rasters are derived from interpolated rasters which are (for most years) constructed from a relatively large number of observations, compared to the Pan_{obs} rasters which are constructed from a relatively small number of observations. It should be noted however that radiation is derived from a range of input variables (Jeffrey *et al.*, 2001), and the number of stations recording those variables (in particular, cloud oktas and sunshine hours) is also declining in recent years. Consequently the benefit of using the synthetic estimate is being eroded. This decline also impacts all other evaporation and evapotranspiration variables provided by SILO ($FAO56$, $ASCE$, M_{pp} , M_{wet} , M_{aa} and M_{lake}) as they all require radiation as an input variable. In recent years, satellite derived estimates of solar radiation have become available, and these estimates are now more

accurate than modelled or interpolation-based methods. However, satellite derived estimates are only available for the last 20-30 years, and discontinuities arise when blending satellite and ground-based estimates. While the satellite estimates of radiation are generally preferred to modelled estimates, it should be noted that in some instances satellite estimates are not available (*e.g.* pixel(s) may be occluded) and/or may be less accurate than modelled or interpolated estimates.

The regression model for synthetic Class A pan was constructed using observational Class A pan data. Pans in different locations have variable exposure to wind and sunlight which can change over time. Consequently, the regression model fitted at a given station may no longer be representative of the local area as the conditions change, resulting in biases in the synthetic estimates. It should also be noted that the regression model does not accurately represent days with very high evaporation, probably due to the lack of daily wind variations in the model, although the same applies to any estimate that does not include wind. For further details, see Rayner *et al.* (2005), Frost *et al.* (2017).

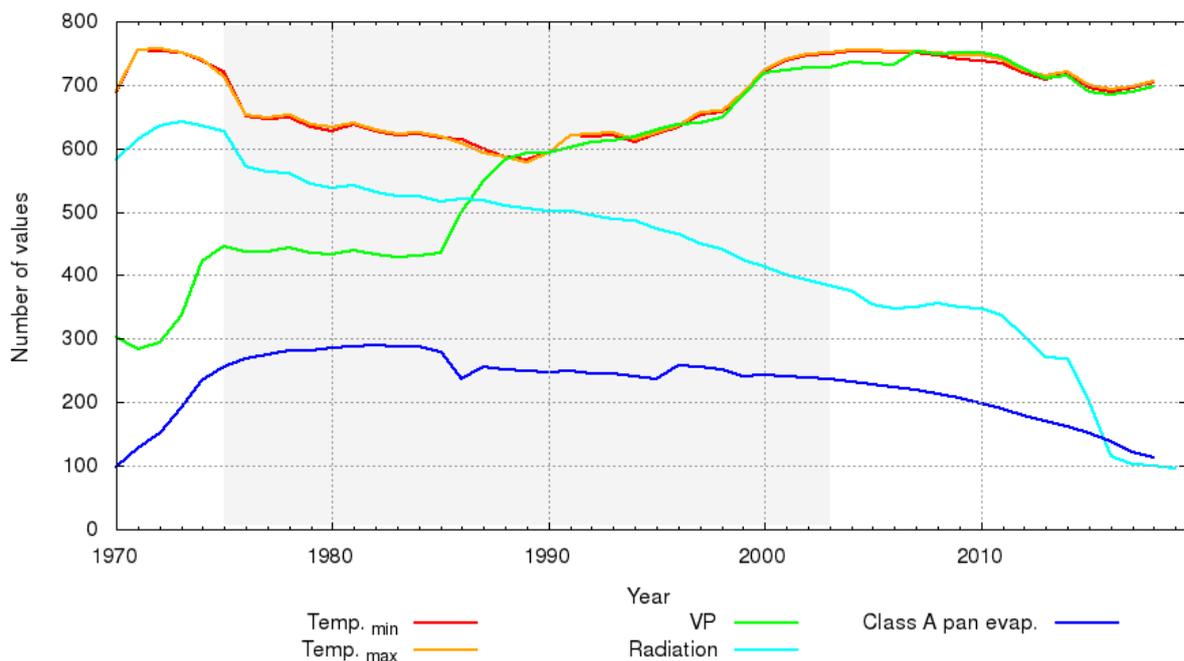


Figure 2. Number of stations within Australia (and surrounding islands) reporting variables used to construct gridded surfaces required for deriving gridded surfaces of synthetic Class A pan evaporation (Pan_{syn}). The number of stations reporting Class A pan evaporation is also shown for comparison. The data shown for radiation are the number of stations for which radiation estimates can be derived from other variables, not the number of stations reporting radiation observations. The years (1975-2003) used to fit the regression model are shaded.

FAO56 is the method recommended by the International Commission for Irrigation and Drainage (ICID) and the Food and Agriculture Organization of the United Nations (FAO) for estimating reference evapotranspiration. It is widely used in irrigation modelling but is also used in many other application areas, including dryland crop modelling. Of all the methods considered, it requires the least empirical parameterization or local calibrations, yet it includes the effect of most climate variables which affect evapotranspiration (solar radiation, wind, and vapour pressure deficit). As a consequence the FAO56 method provides “values that are more consistent with actual crop water use data worldwide” (Allen *et al.*, 1998). SILO recommends the use of FAO56 for irrigation modelling purposes, with the following caveats:

- evapotranspiration in small irrigated fields may be underestimated in dry areas due to advection effects
- SILO estimates of FAO56 currently use a default wind speed of 2 m/s (see Appendix A and further discussion below).

The various Morton models are widely used in hydrological applications because of their flexibility and range of application. For example:

- evaporation over ponds with a small surface area can be computed from M_{pp} and M_{lake} (see Morton, 1983b)
- the point (M_{pp}) and areal (M_{wet}) potential evapotranspiration estimates can be used as an upper limit on actual evapotranspiration, with additional data on soil water availability used to derive actual evapotranspiration
- the M_{aa} model is popular because it is one of relatively few methods that can provide an estimate of actual evapotranspiration without using soil water information and complicated hydrological models
- the M_{lake} model is commonly used in storage level assessment of large dams.

While the Morton's methods are popular, users should however be aware of the following limitations:

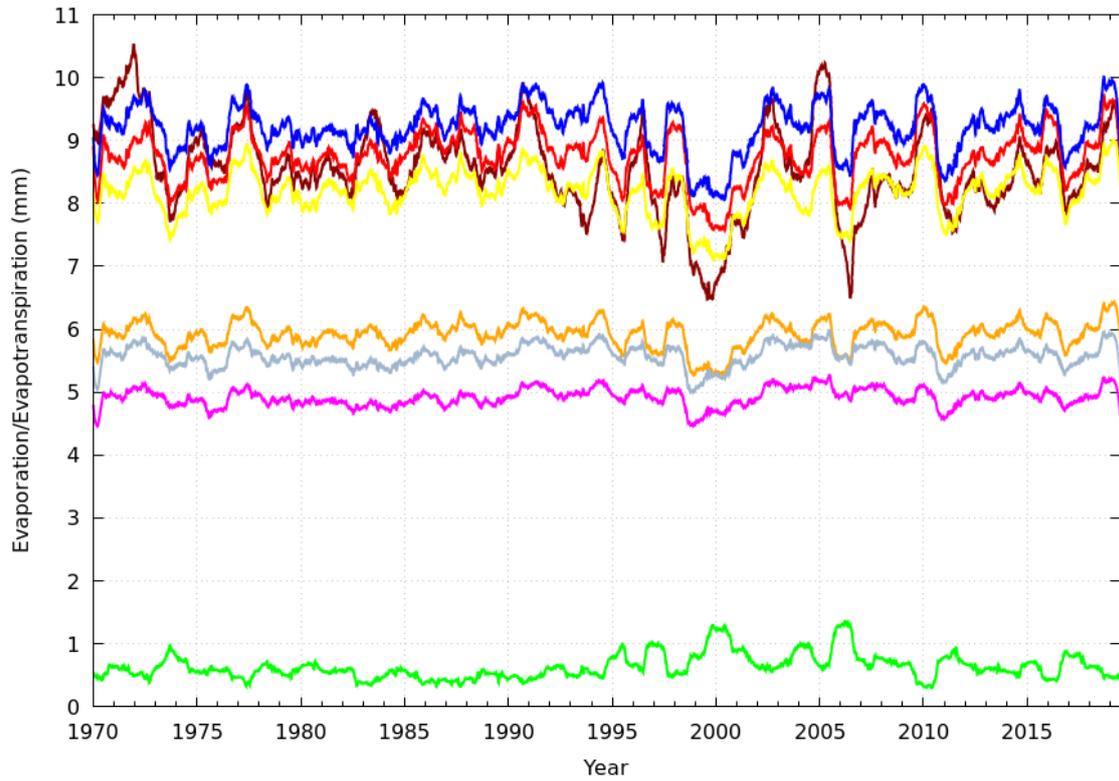
- estimates are more likely to be accurate for accumulation periods of one week or longer
- M_{aa} should only be used as an approximate estimate of actual evapotranspiration when more accurate data are unavailable
- the three areal estimates (M_{wet} , M_{aa} and M_{lake}) should only be used for modelling extended areas, such as relatively large river catchments and lakes
- M_{lake} should only be used for shallow lakes (less than 30 m in depth), or when accumulated to provide annual totals for deep lakes
- Morton's model of areal actual evapotranspiration may generate sequences of zero evapotranspiration in hot, dry areas. While such periods are physically plausible¹, the estimates may not be useful in some applications, such as irrigation scheduling.

Users interested in Class A pan observations may wish to consult Lim *et al.* (2016) for an analysis of factors affecting evaporation. Users interested in evapotranspiration may wish to consult Frost *et al.* (2017) for an evaluation of various estimates, and McMahon *et al.* (2013) for a review of evaporation/evapotranspiration estimates.

4 Data comparison

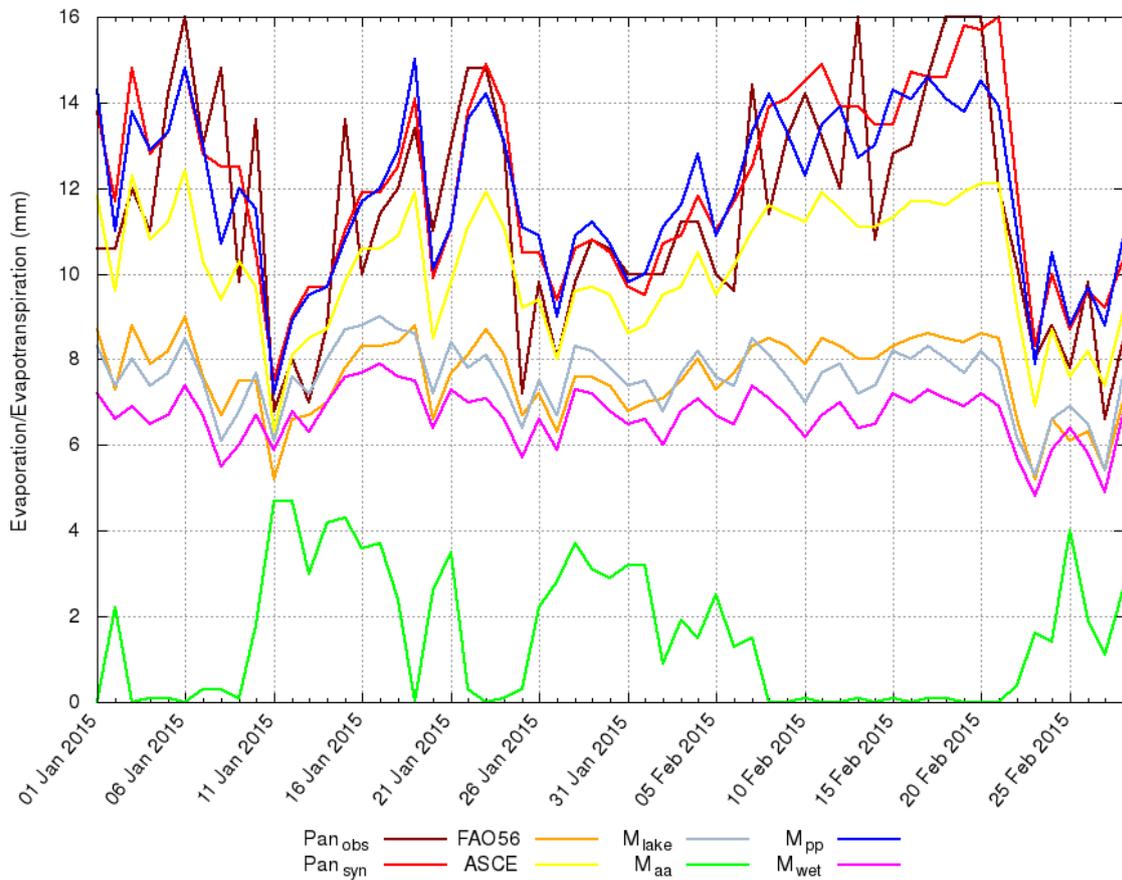
In this section we compare the evaporation and evapotranspiration estimates for selected locations. Figure 3 shows one-year moving averages of the various estimates for Wittenoom (Station 5026) over the period 1 January 1970 – 30 June 2019. There appears to be systematic relationships between some estimates, for example, $FAO56$, M_{lake} and M_{wet} . However, inspection of the daily data for the period 1 January – 28 February 2015 (Figure 4) shows the relationships are not always consistent, which is to be expected given: (i) the diverse nature of the various models; and (ii) the range of factors that affect evaporation and evapotranspiration. It is therefore critical that users select the estimate most suited to their intended application.

¹ For example, dew can prevent evapotranspiration.



Pan_{obs} — FAO56 M_{lake} — M_{pp} — M_{wet} —
 Pan_{syn} — ASCE M_{aa} — M_{wet} —

Figure 3. Comparison of one-year moving averages of the various evaporation and evapotranspiration estimates for Wittenoom (Station 5026, location 118.3358 °E, 22.2425 °S) over the period 1 January 1970 – 30 June 2019.



Pan_{obs} — FAO56 M_{lake} — M_{pp} — M_{wet} —
 Pan_{syn} — ASCE M_{aa} — M_{wet} —

Figure 4. Comparison of the various evaporation and evapotranspiration estimates for Wittenoom (Station 5026, location 118.3358 °E, 22.2425 °S) over the period 1 January – 28 February 2015.

As noted earlier the Penman-Monteith equation requires wind speed as an input variable and this value is currently fixed at 2 m/s in SILO's *FAO56* and *ASCE* datasets. SILO has recently constructed gridded 2m and 10m wind speed datasets for Australia using observational data (Zhang *et al.*, 2020). The effect of using real wind data (instead of the fixed 2 m/s) on the computation of *FAO56* at Wittenoom (Station 5026) is shown in Figure 6. For this location and time period the inclusion of wind data appears to systematically increase the estimated evapotranspiration. The increases arise because the actual 2m wind speed is consistently higher than the default 2m/s over that period. It should however be noted that this behaviour is not always observed. For example, the inclusion of observed wind speed appears to have little impact on the *FAO56* estimates for Townsville (Station 32040) over the same period (Figure 7). In this case there were days when the daily mean wind speed was above (below) 2 m/s, yet there does not appear to be a corresponding increase (decrease) in the *FAO56* estimate on those days. The apparent anomaly can be understood by examining the vapour pressure deficit at the two stations over the given period (see Figure 8). Evapotranspiration was less sensitive to wind at Station 32040 because the relative humidity was comparatively high (as indicated by a low vapour pressure deficit), so the wind did not provide a transport mechanism to drive evapotranspiration. In contrast, the vapour pressure deficit at Station 5026 was relatively high over this period. Consequently the evapotranspiration was increased by wind transporting water vapour away from the evaporating and transpiring surfaces.

The impact of wind on evaporation can be complicated by the presence of updraft. Vertical air movement is more effective at removing moisture from the boundary layer, and therefore drives evaporation significantly more than horizontal air movement.

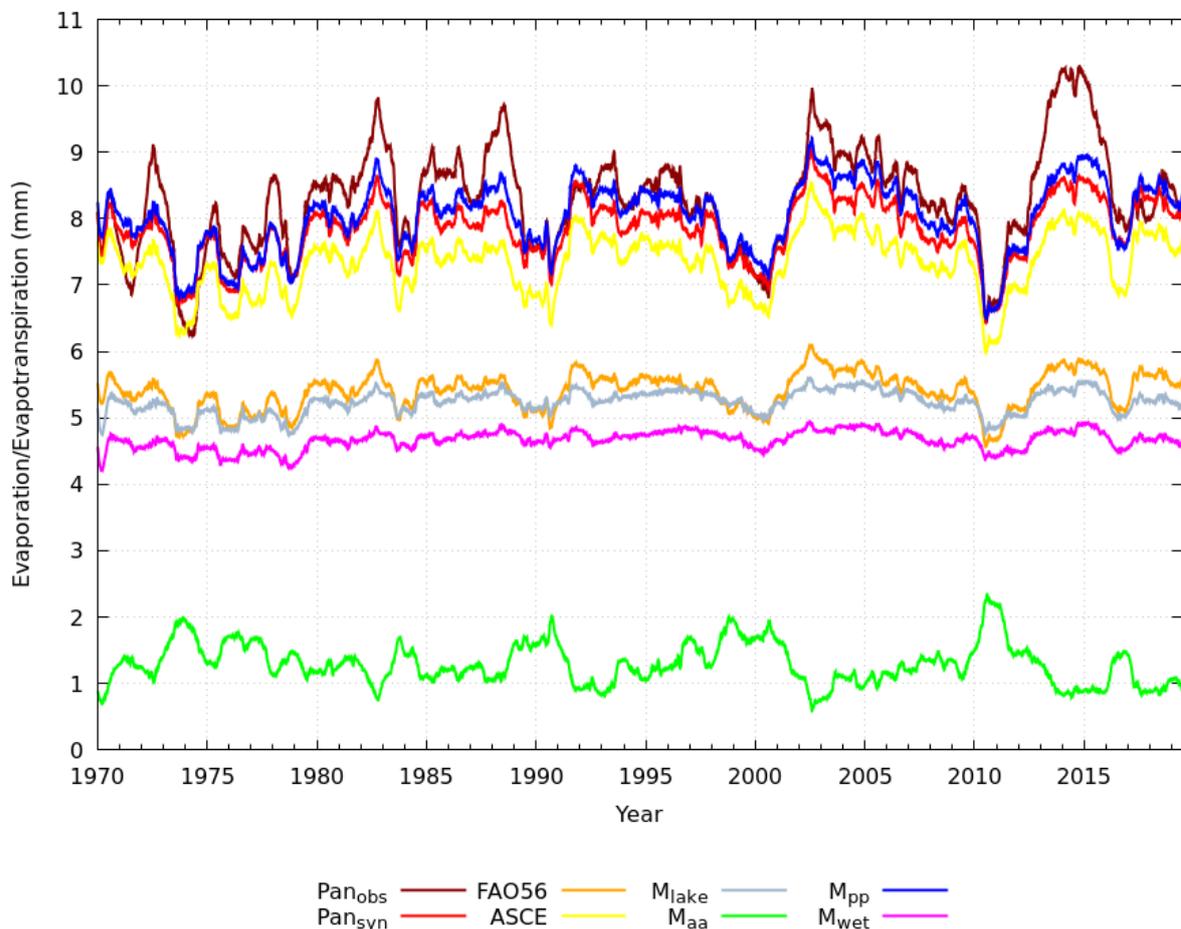


Figure 5. Comparison of one-year moving averages of the various evaporation and evapotranspiration estimates for Longreach (Station 36031, location 144.2828 °E, 23.4397 °S) over the period 1 January 1970 – 30 June 2019.

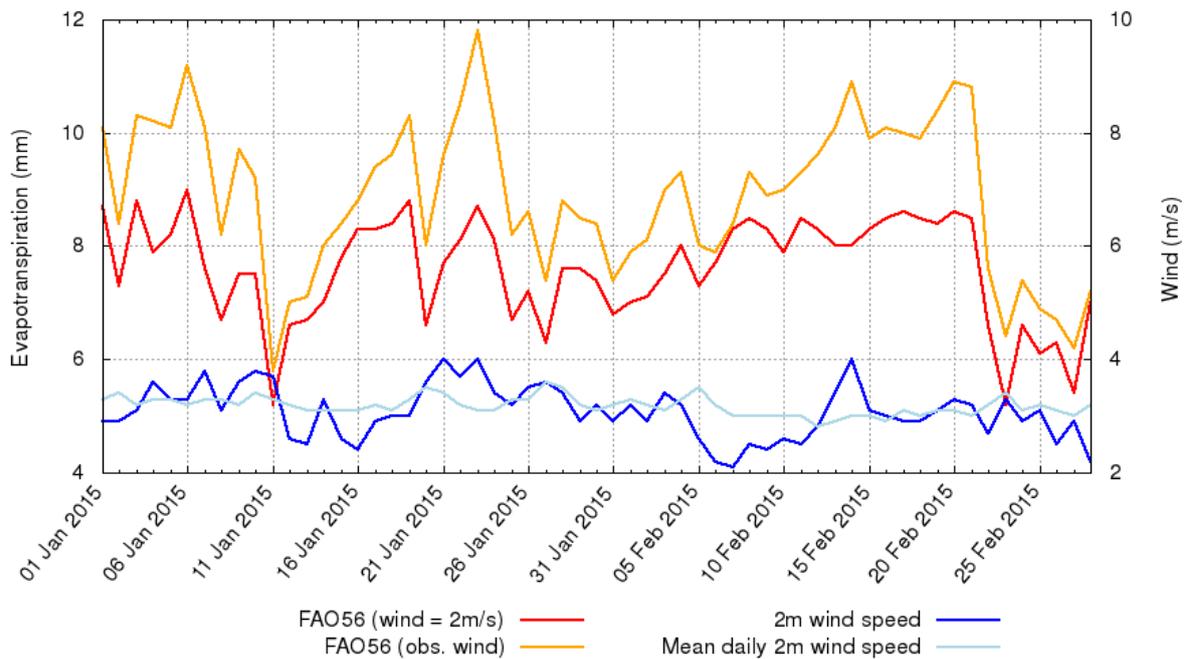


Figure 6. Effect of using a fixed 2 m/s wind speed on the computation of FAO56 evapotranspiration at Wittenoom (Station 5026, location 118.3358 °E, 22.2425 °S) over the period 1 January – 28 February 2015. The mean daily 2m wind speed (1987-2016) is also shown for comparison.

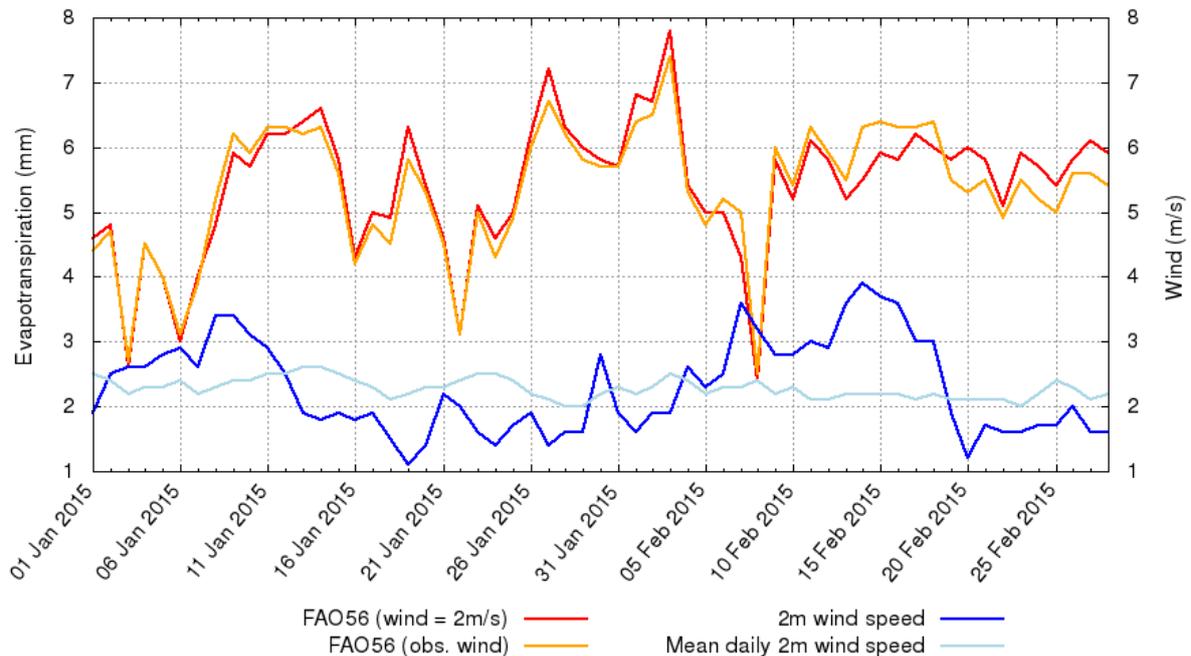


Figure 7. Effect of using a fixed 2 m/s wind speed on the computation of FAO56 evapotranspiration at Townsville (Station 32040, location 146.7661 °E, 19.2483 °S) over the period 1 January – 28 February 2015. The mean daily 2m wind speed (1987-2016) is also shown for comparison.

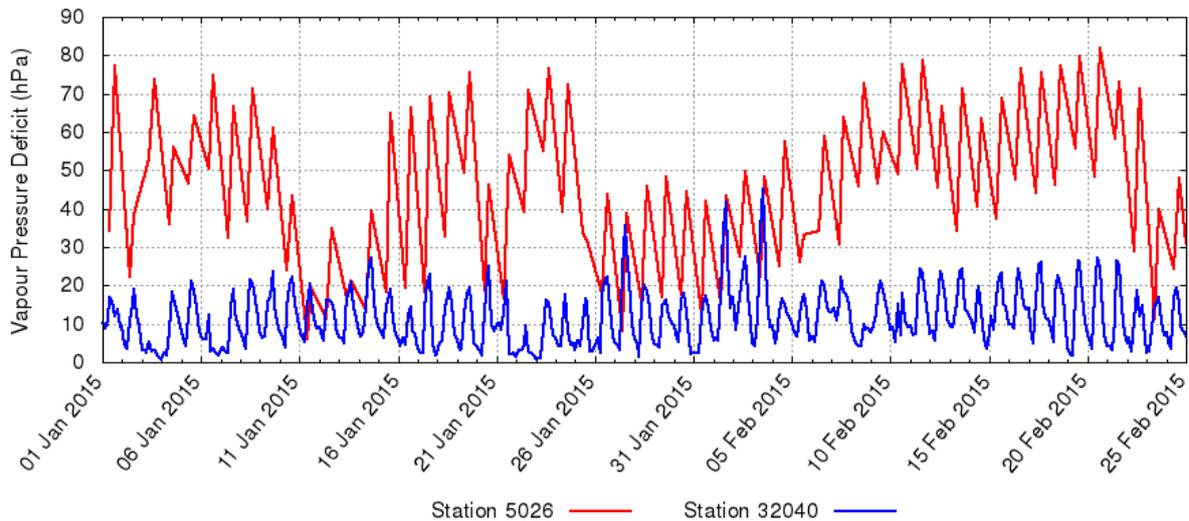


Figure 8. Vapour pressure deficit at Stations 5026 and 32040 over the period 1 January – 28 February 2015. All available hourly observations within the given period are shown.

5 Summary

SILO provides a range of observed and modelled estimates of evaporation and evapotranspiration. This report has provided users with an overview of the various estimates and their respective limitations. Users are urged to carefully consider these limitations in regard to their requirements to ensure the data chosen are fit for purpose.

To improve the potential evaporation and evapotranspiration estimates provided by SILO, future development will focus on the following:

- Recalibration of the synthetic pan model using satellite-derived estimates of radiation. Methods will be investigated for introducing the satellite observations in a manner that will minimise discontinuities occurring when satellite estimates replace observational estimates (for example, when a given station ceases recording cloud oktas or sunshine hours duration).
- Implementation of an anomaly interpolation technique for interpolating Class A pan observations. The anomaly method may provide some resilience to changes in the pan network.
- Introduction of synthetic pan estimates as covariates in the interpolation of Class A pan observations.
- Incorporate the effect of wind on evaporation and evapotranspiration through:
 - a. development of a new model of pan evaporation, using SILO's new wind speed datasets (for example, a model similar to that developed by Lim *et al.* (2016))
 - b. replacement of the fixed 2 m/s wind speed in the computation of *FAO56* and *ASCE* estimates with daily gridded 2m wind speeds. Long term mean 2m wind speed datasets will be used when daily surfaces are unavailable.
- Development of methods for detecting un-flagged accumulations in observational records using independent estimates.
- Introduction of new evaporation/evapotranspiration estimates, such as the PenPan model (Rotstayn *et al.*, 2006) and variations of the original Penman method (in particular, the Shuttleworth (1992) and Priestley–Taylor (1972) methods).

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Appendix A Formulae for computing FAO56 and ASCE evapotranspiration

SILO's FAO56 (short crop) and ASCE (tall crop) estimates of reference evapotranspiration are computed using the Penman-Monteith method, as recommended in Allen *et al.* (1998). The method requires the following input data: Julian day, latitude, elevation, air pressure (station level or mean sea level), minimum and maximum temperatures, solar radiation, 9am vapour pressure and 2m wind speed. The following should be noted:

- wind speed is currently fixed at 2 m/s (see the discussion in the main text)
- solar radiation is estimated using observations of 9am and 3pm cloud oktas, and sunshine hours duration (Zajaczkowski *et al.*, 2013)
- 9am air pressure and 9am vapour pressure are used because daily means are not available. The approximation is justified by the fact that vapour pressure doesn't vary greatly during a day, typically around 10-16% (Jeffrey *et al.*, 2001).

The impact of using the default wind speed and solar radiation derived from cloud-oktas and sunshine hours was reported in Fitzmaurice and Beswick (2005). The default wind speed was found to be the major source of error in the FAO56 calculation.

The Penman-Monteith method uses two coefficients to incorporate the effects of aerodynamic and bulk surface resistance:

- C_n : a function of the timestep and aerodynamic resistance
- C_d : a function of the timestep, bulk surface resistance and aerodynamic resistance.

The FAO56 and ASCE estimates differ only in the values of these two coefficients (see Table 1).

Table 1. Coefficients used in the Penman-Monteith method.

	FAO56	ASCE
C_n	900	1600
C_d	0.34	0.38

A.1 Input data

The input variables are:

T_{max}	daily maximum temperature [°C]
T_{min}	daily minimum temperature [°C]
e_a	calculated daily average vapour pressure [kPa]
R_s	incoming solar radiation [$\text{MJ m}^{-2} \text{day}^{-1}$], derived from cloud oktas and sunshine hours duration
P	observed station-level pressure [kPa],
P_{msl}	mean sea-level pressure [kPa]
u_2	mean wind speed at 2 m height [m/s]

- H station elevation [m],
 φ latitude [radians; negative in the southern hemisphere],
 J Julian day (day number in the year; 1 for 1 Jan, 365 for 31 Dec).

The calculation of e_a and R_s are described in Jeffrey *et al.* (2001).

A.2 The Penman-Monteith equation

In this section we outline the steps required to compute SILO's FAO56 and ASCE estimates. The equation numbers refer to those in Allen *et al.* (1998). The formula for the Penman-Monteith crop reference evapotranspiration is:

$$ET_0 = (0.408 \Delta (R_n - G) + \gamma \frac{C_n}{T_{mean} + 273} u_2 (e_s - e_a)) / (\Delta + \gamma(1 + C_d u_2)) \quad (\text{FAO56, eq. 6})$$

where:

- ET_0 crop reference evapotranspiration [mm day^{-1}]
 R_n net radiation at the crop surface [$\text{MJ m}^{-2} \text{day}^{-1}$]
 G soil heat flux density [$\text{MJ m}^{-2} \text{day}^{-1}$]
 T_{mean} daily mean air temperature at 2 m height [$^{\circ}\text{C}$]
 u_2 daily mean wind speed at 2 m height [m s^{-1}]
 e_s daily mean saturation vapour pressure [kPa]
 e_a daily mean actual vapour pressure [kPa]
 Δ slope of the saturation vapour pressure curve [$\text{kPa } ^{\circ}\text{C}^{-1}$]
 Γ psychrometric constant [$\text{kPa } ^{\circ}\text{C}^{-1}$]
 C_n aerodynamic resistance constant
 C_d bulk surface resistance and aerodynamic resistance constant.

Note that the term $(e_s - e_a)$ is also known as the vapour pressure deficit.

The Penman-Monteith equation can be evaluated using the variables defined below.

The net radiation at the crop surface is:

$$R_n = R_{ns} - R_{nl} \quad (\text{FAO56, eq. 40})$$

where:

- R_n net radiation at the crop surface [$\text{MJ m}^{-2} \text{day}^{-1}$]
 R_{ns} net shortwave radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
 R_{nl} net longwave radiation [$\text{MJ m}^{-2} \text{day}^{-1}$].

The net shortwave radiation is:

$$R_{ns} = (1 - \alpha)R_s \quad (\text{FAO56, eq. 38})$$

where:

- R_{ns} net shortwave radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
 R_s incoming solar radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
 α albedo or surface reflection coefficient (0.23 for the reference grass).

The net longwave radiation is:

$$R_{nl} = \sigma \left[\frac{T_{max,K}^4 + T_{min,K}^4}{2} \right] (0.34 - 0.14 \sqrt{e_a}) \left(1.35 \frac{R_s}{R_{so}} - 0.35 \right) \quad (\text{FAO56, eq. 39})$$

where:

- R_{nl} net longwave radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
- σ Stefan-Boltzmann constant [$4.903 \cdot 10^{-9} \text{ MJ K}^{-4} \text{ m}^{-2} \text{day}^{-1}$]
- $T_{max,K}$ maximum absolute temperature during the 24-hour period [K]
- $T_{min,K}$ minimum absolute temperature during the 24-hour period [K]
- e_a actual vapour pressure [kPa]
- R_s measured solar radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
- R_{so} calculated clear-sky solar radiation [$\text{MJ m}^{-2} \text{day}^{-1}$].

The maximum and minimum absolute temperatures are:

$$T_{max,K} = T_{max} + 273.16$$

$$T_{min,K} = T_{min} + 273.16$$

where:

- T_{max} daily maximum temperature [$^{\circ}\text{C}$],
- T_{min} daily minimum temperature [$^{\circ}\text{C}$].

The clear-sky solar radiation is:

$$R_{so} = (0.75 + 2 \times 10^{-5}h) \quad (\text{FAO56, eq. 37})$$

where:

- R_{so} calculated clear-sky solar radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
- R_a extraterrestrial radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
- h station height [m].

The extraterrestrial radiation is:

$$R_a = \left[\frac{24 \times 60}{\pi} \right] G_{sc} d_r (\omega_s \sin(\varphi) \cos(\delta) + \cos(\varphi) \cos(\delta) \sin(\omega_s)) \quad (\text{FAO56, eq. 21})$$

where:

- R_a extraterrestrial radiation [$\text{MJ m}^{-2} \text{day}^{-1}$]
- G_{sc} solar constant = $0.0820 \text{ MJ m}^{-2} \text{min}^{-1}$
- d_r inverse relative Earth-Sun distance
- ω_s sunset hour angle [rad]
- φ latitude [rad]
- δ solar declination [rad].

Note: R_a can be pre-calculated for each day of the year for a set of fixed latitudes.

The inverse relative Earth-Sun distance:

$$d_r = 1 + 0.033 \cos\left(\frac{2\pi}{365}J\right) \quad (\text{FAO56, eq. 23})$$

where:

- d_r inverse relative Earth-Sun distance
- J Julian day.

The sunset hour angle is:

$$\omega_s = \arccos(-\tan(\varphi) \tan(\delta)) \quad (\text{FAO56, eq. 25})$$

where:

ω_s sunset hour angle [rad]
 φ latitude [rad]
 δ solar declination [rad].

The solar declination is:

$$\delta = 0.409 \sin\left(\frac{2\pi}{365}J - 1.39\right) \quad (\text{FAO56, eq. 24})$$

where:

δ solar declination [rad]
 J Julian day.

The soil heat flux density is set to the recommended default value (Allen *et al.*, 1998):

$$G = 0 \quad (\text{FAO56, eq. 42})$$

where:

G soil heat flux density [$\text{MJ m}^{-2} \text{day}^{-1}$].

The daily mean air temperature is:

$$T_{mean} = \frac{T_{max} + T_{min}}{2} \quad (\text{FAO56, eq. 9})$$

where:

T_{mean} daily mean air temperature at 2 m height [$^{\circ}\text{C}$]
 T_{max} daily maximum temperature [$^{\circ}\text{C}$]
 T_{min} daily minimum temperature [$^{\circ}\text{C}$].

The daily mean wind speed is currently set to a default value:

$$u_2 = 2.0 \quad (\text{FAO56, p. 63})$$

where:

u_2 daily mean wind speed at 2 m height [m s^{-1}].

The daily mean saturation vapour pressure is:

$$e_s = \frac{e_s(T_{max}) + e_s(T_{min})}{2} \quad (\text{FAO56, eq. 12})$$

where:

e_s daily mean saturation vapour pressure [kPa].

The saturation vapour pressure at air temperature T [$^{\circ}\text{C}$] is:

$$e_s(T) = 0.6108e^{\left(\frac{17.27T}{T+237.3}\right)}. \quad (\text{FAO56, eq. 11})$$

The slope of the saturation vapour pressure curve is:

$$\Delta = \frac{4098 \left[0.6108 e^{\left(\frac{17.27 T_{mean}}{T_{mean} + 237.3} \right)} \right]}{(T_{mean} + 237.3)^2} \quad (\text{FAO56, eq. 13})$$

where:

Δ slope of the saturation vapour pressure curve [kPa °C⁻¹] at T_{mean}
 T_{mean} daily mean air temperature at 2 m height [°C].

The psychrometric constant is:

$$\gamma = 0.665 \times 10^{-3} p \quad (\text{FAO56, eq. 8})$$

where:

γ psychrometric constant times pressure [kPa °C⁻¹]
 p station-level pressure [kPa].

If observed station-level pressure is not available, it is calculated from interpolated mean sea-level pressure using:

$$P = P_{msl} - 1.195 \times 10^{-2} h + 5.1681 \times 10^{-7} h^2 \quad (\text{Jeffrey et al., eq. 5})$$

where:

P station-level pressure [kPa]
 P_{msl} mean sea-level pressure [kPa]
 h station height [m].

Appendix B Formulae for computing Morton's evaporation and evapotranspiration

SILO computes Morton's estimates of evaporation and evapotranspiration as follows:

- point potential evapotranspiration, M_{pp}
- areal actual evapotranspiration, M_{aa}
- wet environment areal potential evapotranspiration, M_{wet}
- shallow lake evaporation, M_{lake} .

SILO uses the methodology outlined in Morton (1983a), with minor modifications as noted below.

B.1 Input data

The input variables are:

T_{max}	daily maximum temperature [°C]
T_{min}	daily minimum temperature [°C]
e_a	vapour pressure at 9am [kPa]
R_s	incoming solar radiation [$\text{MJ m}^{-2} \text{day}^{-1}$], derived from cloud oktas and sunshine hours duration
φ_d	latitude [degrees; negative in the southern hemisphere]
H	elevation above mean sea level [m].

The date is also required (for calculating the extra-terrestrial solar radiation).

B.2 Overview

Morton's point potential evapotranspiration is based on two equations:

$$M_{pp} = R_T - \left[\gamma_p f_T + 4\varepsilon\sigma(T_p + 273)^3 \right] (T_p - T) = R_T - \lambda_p f_T (T_p - T)$$

(Morton eq. 10)

$$M_{pp} = f_T (v_p - v_d)$$

(Morton eq. 11)

where:

M_{pp}	point potential evapotranspiration in units of latent heat
T_p, T	equilibrium and air temperatures, respectively
R_T	net radiation for land surface at air temperature
f_T	vapour transfer coefficient
γ_p	psychrometric constant, equivalent to γp in Equation 10 (Morton, 1983a); the same as γ in the FAO56 Penman-Monteith equation
σ	Stefan-Boltzmann constant
ε	surface emissivity
v_p	saturation vapour pressure at T_p
v_d	saturation vapour pressure at the dew point temperature
λ_p	heat transfer coefficient as $\lambda_p = \gamma_p + 4\varepsilon\sigma(T_p + 273)^3 / f_T$.

Morton's wet environment areal potential evapotranspiration, M_{wet} is calculated from net solar radiation using an empirical function:

$$M_{wet} = b_1 + b_2 \left(1 + \frac{\gamma_p}{\Delta_p}\right)^{-1} R_{TP} \quad (\text{Morton eq. 14})$$

where:

b_1, b_2 empirical constants
 Δ_p slope of saturation vapour pressure at T_p
 R_{TP} R_T at T_p .

Morton's areal actual evapotranspiration, M_{aa} can then be calculated using the complementary relationship:

$$M_{pp} + M_{aa} = 2 M_{wet} \quad (\text{Morton eq. 8})$$

Morton's shallow lake evaporation, M_{lake} , is calculated using an expression similar to that used above for M_{wet} , but with different input data (see Section B.4).

B.3 Procedure for calculating Morton's evapotranspiration over land: M_{pp} , M_{aa} and M_{wet}

Morton (1983a) provided an algorithm for calculating monthly values. SILO computes daily values using a modified version of Morton's original method. Users should note that Morton used sunshine hours to estimate solar radiation, while SILO uses cloud oktas and sunshine hours duration (Zajackowski *et al.*, 2013). Furthermore, SILO uses a method adapted from Allen *et al.*, (1998) to compute the extraterrestrial radiation (see Step 5, below).

The procedure² for computing M_{pp} , M_{aa} and M_{wet} is given below. Equations labelled with "Morton" refer to those in Morton (1983a) and those labelled with "FAO56" refer to those in Allen *et al.*, (1998).

Step 0. Convert units

$\varphi = \varphi_a \pi / 180$ (Convert degrees to radians)
 $e_a = 10 e_{a,kPa}$ (Convert kPa to mbar. Note: 1 mbar = 1 hPa)
 $G = R_s / 0.0864$ (Convert MJ m⁻² day⁻¹ to Wm⁻²)

Step 1. Calculate the ratio of station-level pressure to mean sea level pressure

$$r_p = \frac{p}{p_s} = \left(\frac{288 - 0.0065h}{288}\right)^{5.256} \quad (\text{Morton eq. C-1})$$

where:

p station level atmospheric pressure
 p_s mean sea level atmospheric pressure
 h elevation above mean sea level [m].

Step 2. Calculate the daily mean temperature using maximum and minimum air temperatures

² The procedure is organized in a manner that will assist users in following the computation of intermediate variables; it is not arranged in the same order as the original algorithm in Morton (1983a).

$$T_a = \frac{T_{max} + T_{min}}{2}$$

Step 3. Calculate the psychrometric constant, γ_p [mbar/ °C]

$$\begin{aligned} \gamma_p &= 0.066r_p & \text{if } T_a \geq 0^\circ\text{C} \\ \gamma_p &= 0.0574r_p & \text{if } T_a < 0^\circ\text{C} \end{aligned} \quad (\text{Morton, eq. C-31, FAO56, eq. 8})$$

Note: Morton defines psychrometric constant (γ) as $c_p/\epsilon\lambda$ and uses (γp) in computation. In Allen *et al.* (1998), psychrometric constant (γ) is defined as $c_p p/\epsilon\lambda$, with the pressure term included. In the above equations subscript p in γ_p indicates that the pressure term is included.

Step 4. Calculate the saturation vapour pressure and the slope of the saturation vapour pressure/temperature curve at air temperature T_a

(1) saturation vapour pressure (mbar):

$$\begin{aligned} e_s(T_a) &= 6.108 \exp\left(\frac{17.27T_a}{T_a+237.3}\right) & \text{if } T_a \geq 0^\circ\text{C} \\ e_s(T_a) &= 6.108 \exp\left(\frac{21.88T_a}{T_a+265.5}\right) & \text{if } T_a < 0^\circ\text{C} \end{aligned} \quad (\text{Morton, eq. C-4, FAO56, Eq. 3-11})$$

The above calculation can be improved³:

$$e_s = \frac{1}{4}e_s(T_{max}) + \frac{1}{4}e_s(T_{min}) + \frac{1}{2}e_s(T_{mean})$$

or as in Allen *et al.* (1998):

$$e_s = \frac{1}{2}e_s(T_{max}) + \frac{1}{2}e_s(T_{min})$$

(2) slope of the saturation vapour pressure / temperature curve (mbar/°C):

$$\begin{aligned} \Delta &= \frac{4098e_s}{(T_a+237.3)^2} & \text{if } T_a \geq 0^\circ\text{C} \\ \Delta &= \frac{5809e_s}{(T_a+265.5)^2} & \text{if } T_a < 0^\circ\text{C} \end{aligned} \quad (\text{Morton, eq. C-5, FAO56, eq. 3-13})$$

Step 5. Calculate the extraterrestrial radiation, G_E , (shortwave radiation incident upon the earth's atmosphere). It is a function of the location and time of the year.

$$G_E = \frac{24 \times 60}{\pi} G_{sc} d_r [\omega_s \sin(\varphi) \sin(\delta) + \cos(\varphi) \cos(\delta) \sin(\omega_s)] / 0.0864 \quad (\text{FAO56, eq.3-21})$$

where:

- G_E extraterrestrial radiation [Wm^{-2}]
- G_{sc} solar constant ($0.0820 \text{ MJ m}^{-2} \text{ min}^{-1}$)
- φ latitude [radians]

The constant 0.0864 used in the above equation is for converting the units from $\text{MJ m}^{-2} \text{ d}^{-1}$ to Wm^{-2}

The inverse relative Earth-Sun distance, d_r , is given by:

³ Francis Chiew, *pers comm.*

$$d_r = 1 + 0.033 \cos(2\pi J / 365) \quad (\text{FAO56, eq. 3-23})$$

The solar declination δ is given by:

$$\delta = 0.409 \sin(2\pi J / 365 - 1.39) \quad (\text{FAO56, eq. 3-24})$$

where J is the Julian day (day number in the year; 1 for 1 Jan, 365 for 31 Dec).

The solar angle between sunrise and noon ω_s (same as sunset angle in Allen *et al.*, 1998) is given by:

$$\omega_s = \arccos [-\tan(\varphi) \tan(\delta)] \quad (\text{FAO56, eq. 3-25})$$

where ω_s is the same as the (apparently) more complex formula provided by Morton (1983a).

Step 6. Compute the relative sunshine duration (S), and the transmittance and absorption of solar radiation by clear sky. Sunshine duration is used later to calculate albedo and net longwave radiation.

(1) Estimate precipitable water vapour (W) in millimetres and a turbidity coefficient (j)

$$w = e_a / (0.49 + T_a / 129) \quad (\text{Morton, eq. C-16})$$

$$j = (0.5 + 2.5 \cos^2(z)) \exp[c_1(r_p - 1)] \quad (\text{Morton, eq. C-18})$$

where $c_1 = 21 - T_a$ (Morton eq. C-17), and subject to the constraint $0 \leq c_1 \leq 5$ (Morton eq. C-17a), and

$$\cos(z) = \cos(\phi - \delta) + (\sin(\omega_s) / \omega_s - 1) \cos(\varphi) \cos(\delta)$$

(2) Compute the transmittance of clear skies to direct beam solar radiation (τ) and estimate the part of τ that is the result of absorption (τ_a)

$$\tau = \exp\left(-0.089 \left(\frac{r_p}{\cos(z)}\right)^{0.75} - 0.083 \left(\frac{j}{\cos(z)}\right)^{0.90} - 0.029 \left(\frac{W}{\cos(z)}\right)^{0.60}\right) \quad (\text{Morton, eq. C-19})$$

$$\tau_a = \exp\left(-0.0415 \left(\frac{j}{\cos(z)}\right)^{0.90} - 0.0029^{0.5} \left(\frac{W}{\cos(z)}\right)^{0.30}\right) \quad (\text{Morton, eq. C-20})$$

subject to the constraint:

$$\tau_a \geq \exp\left(-0.0415 \left(\frac{j}{\cos(z)}\right)^{0.90} - 0.029 \left(\frac{W}{\cos(z)}\right)^{0.60}\right) \quad (\text{Morton, eq. C-20a})$$

(3) Compute the clear-sky global radiation

$$G_0 = G_E \tau \left[1 + \left(1 - \frac{\tau}{\tau_a}\right) (1 + a_0 \tau)\right] \quad (\text{Morton, eq. C-21})$$

(4) Compute the relative sunshine duration

$$S = 0.53G / (G_0 - 0.47G) \quad (\text{Morton, eq. 43})$$

with $0 \leq S \leq 1.0$.

Step 7. Calculate albedo

(1) Zenith value of the dry-season snow-free clear-sky albedo

$$a_{zd} = 0.26 - 0.00012P_A r_p^{0.5} [1 + |\phi_d/42| + (\phi_d/42)^2] \quad (\text{Morton, eq. C-2})$$

where:

 P_A annual precipitation [mm] ϕ_d latitude [degrees]subject to the constraint: $0.11 \leq a_{zd} \leq 0.17$ (Morton, eq. C-2a)

(2) Zenith value of snow-free clear-sky albedo

$$a_{zz} = a_{zd} \quad (\text{Morton, eq. C-12})$$

subject to the constraint: $0.11 \leq a_{zz} \leq 0.5(0.91 - e_a/e_s)$.

(3) Zenith value of clear-sky albedo

$$a_z = a_{zz} + (1 - c_0^2)(0.34 - a_{zz}) \quad (\text{Morton, eq. C-14})$$

where $c_0 = e_s - e_a$ and subject to $0 \leq c_0 \leq 1$.

(4) Clear-sky albedo

$$a_0 = \frac{a_z \{ \exp(1.08) - [\frac{2.16 \cos(\phi - \delta)}{\pi} + \sin(\phi - \delta)] \exp[0.012(\phi - \delta) * \frac{180}{\pi}] \}}{1.473[1 - \sin(\phi - \delta)]} \quad (\text{Morton, eq. C-15})$$

(5) Albedo

$$a = a_0 \left[S + (1 - S) \left(1 - \frac{(\phi - \delta) * \frac{180}{\pi}}{330} \right) \right] \quad (\text{Morton, eq. C-23})$$

Step 8. Calculate net incoming solar or shortwave radiation, R_{ns}

$$R_{ns} = (1 - \alpha) G \quad (\text{FAO56, eq. 3-38})$$

where:

 R_{ns} is the net incoming solar radiation [Wm^{-2}] and α is the albedo.**Step 9. Calculate net outgoing longwave radiation, R_{nl}** (1) estimate the proportional increase in atmospheric radiation due to clouds (ρ)

$$\rho = 0.18[(1 - c_2)(1 - S)^2 - c_2(1 - S)^{0.5}]/r_p \quad (\text{Morton, eq. C-25})$$

where:

$$c_2 = 10(e_a/e_s - S - 0.42) \text{ and subject to } 0 \leq c_2 \leq 1. \quad (\text{Morton, eq. C-24})$$

(2) estimate R_{nl}

$$R_{nl} = \varepsilon \sigma (T_a + 273)^4 [1 - (0.71 + 0.007e_a r_p)(1 + \rho)] \quad (\text{Morton, eq. C-26})$$

subject to the constraint

$$R_{nl} \geq 0.05 \varepsilon \sigma (T_a + 273)^4$$

Step 10. Calculate net radiation, R_n [$\text{MJ m}^{-2} \text{d}^{-1}$]

$$R_n = R_{ns} - R_{nl} \quad (\text{FAO56, eq.3-40})$$

Step 11. Calculate the stability factor (ξ), vapour pressure transfer coefficient (f_a) and heat transfer coefficient (λ)

(a) Calculate the stability factor (ξ)

(a-1) calculate $e_s - e_a$ and if less than 0, set $e_s - e_a = 0.0001$

(a-2) calculate

$$\xi = \left[0.28 \left(1 + \frac{e_a}{e_s} \right) + \frac{\Delta R_{tc}}{\gamma_p \left(\frac{1}{r_p} \right)^{0.5} f_z b_0 (e_s - e_a)} \right]^{-1}$$

where $R_{tc} = R_n$, subject to $R_{tc} > 0$, and $b_0 = 1.0$

(a-3) apply constraint: if $\xi > 1.0$, then set $\xi = 1.0$.

In (a-1) to (a-3):

e_a = actual vapour pressure (daily average)

e_s = saturation vapour pressure (daily average)

Δ = slope of the saturation vapour pressure / temp curve [$\text{mbar}/^\circ\text{C}$]

R_n = net radiation [Wm^{-2}]

γ_p = psychrometric constant [$\text{mbar}/^\circ\text{C}$]

$r_p = p/p_s$, the ratio of station level pressure to mean sea level pressure

$f_z = 28.0 \text{ Wm}^{-2} \text{ mbar}^{-1}$ if $T_a \geq 0^\circ\text{C}$, and $f_z = 28.0 * 1.15 \text{ Wm}^{-2} \text{ mbar}^{-1}$ if $T_a < 0^\circ\text{C}$.

(b) Calculate the vapour transfer coefficient (f_T)

$$f_T = (1/r_p)^{0.5} f_z / \xi \quad (\text{Morton, eq. C-30})$$

(c) Calculate the heat transfer coefficient (λ)

$$\lambda = \gamma_p + (4\varepsilon\sigma(T_a + 273)^3) / f_T \quad [\text{mbar}/^\circ\text{C}] \quad (\text{Morton, eq. C-31})$$

The value of $\varepsilon\sigma$ is $5.22 \cdot 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$.

Step 12. Iteratively calculate potential evaporation equilibrium quantities, such that the energy balance is satisfied.

(a) Initialise

$$e_p^0 = e_s(T_a)$$

$$T_p^0 = T_a$$

$$\Delta_p^0 = \Delta(T_a)$$

(b) Iterate:

1. Calculate the temperature increment:

$$\delta T = \left(\frac{R_n}{f_a} + e_a + \lambda(T_a - T_p^0) - e_p^0 \right) / (\Delta_p^0 + \lambda) \quad (\text{Morton, eq. C-32})$$

2. Update the equilibrium quantities:

$$T_p = T_p^0 + \delta T \quad (\text{Morton, eq. C-33})$$

$$e_p = e_s(T_p) \quad (\text{Morton, eq. C-34})$$

$$\Delta_p^0 = \Delta(T_p) \quad (\text{Morton, eq. C-35})$$

If the absolute value of δT is less than 0.01°C, exit the loop; otherwise, continue.

3. Update the initial values

$$e_p^0 = e_p$$

$$T_p^0 = T_p$$

$$\Delta_p^0 = \Delta_p$$

Step 13. Calculate Morton's point potential evapotranspiration, M_{pp} in $\text{Wm}^{-2}\text{day}^{-1}$:

$$M_{pp} = R_n - \lambda f_T (T_p - T_a) \quad (\text{Morton, eq. C-36})$$

Step 14. Calculate Morton's wet environment areal potential evapotranspiration, M_{wet} in $\text{Wm}^{-2}\text{day}^{-1}$:

- (a) Calculate net radiation that would occur at the potential evaporation equilibrium temperature (in $\text{Wm}^{-2}\text{day}^{-1}$):

$$R_{np} = M_{pp} + \gamma_p f_T (T_p - T_a) \quad (\text{Morton, eq. C-37})$$

- (b) Calculate M_{wet} :

$$M_{wet} = b_1 + b_2 \left(1 + \frac{\gamma_p}{\Delta_p} \right)^{-1} R_{np} \quad (\text{Morton, eq. C-38})$$

where $b_1 = 14/11.574$ and $b_2 = 1.2$.

- (c) M_{wet} is then modified to satisfy:

$$\frac{1}{2} M_{pp} \leq M_{wet} \leq M_{pp}. \quad (\text{Morton, eq. C-38a})$$

Step 15. Calculate Morton's areal actual evapotranspiration, M_{aa} in $\text{Wm}^{-2}\text{day}^{-1}$:

$$M_{aa} = 2M_{wet} - M_{pp} \quad (\text{Morton, eq. C-39})$$

Step 16. Convert the units from $\text{Wm}^{-2}\text{day}^{-1}$ to mm day^{-1} :

$$M_{pp} = M_{pp} / 28.5$$

$$M_{wet} = M_{wet} / 28.5$$

$$M_{aa} = M_{aa} / 28.5$$

B.4 Procedure for calculating Morton's evaporation over shallow lakes: M_{lake}

Morton (1983a) provided a method for calculating the evaporation over shallow lakes (depth less than 30 m) from measurements at nearby land-based meteorological stations. The method is almost identical to that used for computing wet environment areal evapotranspiration over land (M_{wet}):

$$M_{lake} = b_1 + b_2(1 + \frac{\gamma}{\Delta_p})^{-1}R_{TP}$$

however b_1 and b_2 take slightly different values, and R_{TP} is the net radiation at the equilibrium temperature over water (water has, on average, a smaller albedo than land). To calculate the equilibrium temperature, the following equations are solved:

$$M_{pp} = R_T - [\gamma_p f_T + 4\epsilon\sigma(T_p + 273)^3](T_p - T) = R_T - \lambda_p f_T(T_p - T)$$

$$M_{pp} = f_T(v_p - v_D)$$

These equations are identical to those used for computing Morton's point potential evapotranspiration over land (Morton's equations 10 and 11, shown above in Section B.4). However in this case some coefficients take different values and M_{pp} is Morton's point potential evaporation over *water*, but in a land environment (evaporation from a point water source in the middle of vast land).

The procedure for calculating M_{lake} is the same as that used for calculating evapotranspiration over land (see Section B.3), with the following modifications:

1. Set $a_z = 0.05$ in Step 7.2 (after it has been calculated for the land option)
2. Set $\epsilon\sigma = 5.5 \cdot 10^{-8}$, before it is used in Step 9.2
3. Set $f_z = 25$ for $T_a \geq 0^\circ\text{C}$ and $f_z = 25 \cdot 1.15$ for $T_a < 0^\circ\text{C}$, in Step 11(b)
4. Set $b_0 = 1.12$ in Step 11(a).2
5. Set $b_1 = 13$ and $b_2 = 1.12$ in Step 14(b)
6. Set $M_{aa} = 0$ at Step 15 (after it has been calculated for the land option).

Appendix C Source code

SILO's *FAO56*, *ASCE* and Morton's estimates of evaporation and evapotranspiration can be computed using the source code provided below.

```

// The following references were used throughout the code:
//
// 1. Allen, R.G., Pereira, L.S., Raes, D., and Smith M. 1998: Crop evapotranspiration - Guidelines for
//    computing crop water requirement. FAO Irrigation and Drainage Paper 56,
//    by Food and Agriculture Organization of the United Nations, 300 pp.
//
// 2. ASCE's Standardized Reference Evapotranspiration Equation, proceedings of the National Irrigation
//    Symposium, Phoenix, Arizona, 2000.
//
// 3. Morton, F. I., 1983 (a): Operational estimates of areal evapotranspiration and their
//    significance to the science and practice of hydrology. J. Hydrol., 66, 1-76.
//
// 4. Morton, F. I., 1983 (b): Operational estimates of lake evaporation. Journal of Hydrology, 66, 77-100.

#include <stdio.h>
#include <math.h>
#include <stdlib.h>

// Definitions:
#define fao56_crop 1
#define tall_crop 2
#define ascepm_crop tall_crop

double Es_hPa(const double Temp){
    // Saturated Vapour Pressure in hPa
    return 0.6108*exp(17.27*Temp/(Temp+237.3)); //FAO56 p36 eq 11
}

double Es_mbar(const double Temp){
    // Saturated Vapour Pressure Es function in mbar units
    if (Temp >= 0.0 ) {
        // From https://www.agric.gov.ab.ca/acis/docs/mortons/mortons-evaporation-report.pdf, p109 (5)
        return 6.108 * exp(17.27*Temp/(Temp+237.3));
    }else{
        // above: FAO56 p36 eq 11, but with unit conversion
        return 6.108 * exp(21.88*Temp/(Temp+265.5));
    }
}

```

```

}

double BigDeltaFunction(const double Temp) {
    // Slope of Saturation Vapor Pressure unit of mbar.K-1 (mbar is same as hPa)
    // from FAO56 Chapter 3, Air Humidity equation 13

    if ( Temp >= 0 ) {
        return 4098.0 * Es_mbar(Temp)/pow((Temp+237.3),2.0); // FAO56 Chapter 3 eq. 13
    }else{
        return 5809.0 * Es_mbar(Temp)/pow((Temp+265.5),2.0);
    }
}

double pet_fao56(const double Tmax, const double Tmin, const double SolarR, const double VapourP_hpa,
                const double WSpeed2, const double PSeaIn, const double PStationIn, const double Height,
                const double Lat, const long MyDate, const long DOY, const long crop_height_flag){

    // Function calculates the daily potential evapotranspiration (ETp) using the Penman-Monteith
    // equation as recommended in the book 'Crop Evapotranspiration' published by World Food and
    // Agriculture Organization (FAO) as FAO56 where appropriate.
    //
    // Notes:
    //     1 use default wind = 2 m/s if not available at all
    //     2 no pressure measurement required
    //     3 use PSea=0 when unavailable, use PStation=0 when unavailable too
    //     3-1 when PSea=0 and PStation=0, the program will use default PSea
    //         and Height to calculate PStation
    // Parameters:
    // * double Tmax, daily maximum temperature, in degree of celcius
    // * double Tmin, daily minimum temperature, in degree of celcius
    // * double SolarR, solar radiation, in MJ per squre-meter per day; MJ is Mega Joul
    // * double VapourP_hpa, average vapor pressure at air temperature, in hPa
    // * double WSpeed2, 2m wind speed m/s (set WSpeed2=2 when wind data are not available)
    // * double Height, station height, in meter
    // * double PSeaIn, input sea-level pressure, in hPa
    // * double PStationIn, input station pressure, in hPa
    // * double Lat, latitude, in degree

```

```

//      * long      MyDate, eight-digit date as interger in yyyymmdd format
//      * long      crop_height_flag, flag indicates if the tall or short crop version (FAO56 or ASCEPM) is required
// Returns:
//      daily potential evapotranspiration

double      result;
double      LatR;
double      Ea, Rs, PSea, PStation, Gamma, Tmean, BigDelta, VapourP, VapourP_S;
double      SolarDelination, SunSetHour, SunDistanceInverse;
double      ET_SolarR, ClearSky_SolarR, LongWaveR_Net, SolarR_net, RadiationNet;
double      Temp, temp1;
double      vpd;

const double Pai = 3.1415926535;
const double Gsc = 0.08165;
const double Albedo = 0.23;           // For hypothetical grass reference crop, see equation 3-38
double Sigma = 4.903 * pow(10.0,-9.0); // Stefan Boltzmann
VapourP = VapourP_hpa/10.0;

// Calculate Station Level Pressure
PStation = PStationIn / 10.0;
PSea      = PSeaIn / 10.0;
if (PStation < 0.1) {
    if (PSea < 0.1) {
        PSea=101.3;
    }
    PStation=PSea*pow( ( (293.0 - 0.0065*Height)/293.0 ), 5.26);
}

// -----C1: Gamma, temperature and vapor pressure -----
// Calculate Psychrometric Constant, P32 eq 8
Gamma = 0.665 * 0.001*PStation;

// Calculate daily mean temperature, P33 eq 9
Tmean = (Tmax+Tmin)/2.0;

// Calculate slope of saturation vapor pressure vs temperature, FAO56 p37 eq 13

```

```

BigDelta = 4098.0 * 0.6108 * exp(17.27*Tmean/(Tmean+237.3)) /pow((Tmean+237.3),2.0);

// Calculate Saturation Vapor Pressure, FAO56 p36 eq 12
VapourP_S = (Es_hPa(Tmax)+Es_hPa(Tmin))/2.0;

//-----C2: radiation -----
LatR=Lat/180.0*Pai;

// Calculate net longwave radiation
SolarDelination = 0.409 * sin( 2.0 * Pai * (double)DOY/365.0 - 1.39); // FAO56 p46 eq 24
SunSetHour = acos ( -tan(LatR) * tan(SolarDelination) ); // FAO56 p46 eq 25
SunDistanceInverse = 1.0 + 0.033 * cos( 2.0*Pai /365.0 * (double)DOY ); // FAO56 p46 eq 23

ET_SolarR = 24.0 * 60.0/Pai * Gsc * SunDistanceInverse; // FAO56 p46 eq 21
ET_SolarR = ET_SolarR * ( SunSetHour * sin(LatR) * sin(SolarDelination) // as above
+ cos(LatR) * cos(SolarDelination) * sin(SunSetHour) );

// Clear sky assumes low pollution and low dust
ClearSky_SolarR = ( 0.75 + 2.0*0.00001*Height ) * ET_SolarR;

temp1 = SolarR / ClearSky_SolarR;
if ( temp1 > 1.0 ) {
    temp1 = 1.0;
}

LongWaveR_Net = ( 0.34 - 0.14 * sqrt(VapourP) ) * (1.35 * temp1 - 0.35 ); // FAO56 p52 eq 39
LongWaveR_Net = LongWaveR_Net * Sigma * ( pow((Tmax + 273.16), 4.0) + pow((Tmin + 273.16), 4.0) ) / 2.0;

// Calculate net shortwave radiation
SolarR_net = SolarR * ( 1.0 - Albedo ); // p51 eq 38

// Calculate total net radiation
RadiationNet = SolarR_net - LongWaveR_Net; // p53 eq 40

// -----C3: PET (p24 eq 6)
vpd = VapourP_S - VapourP;
if ( vpd < 0 ) vpd = 0;

```

```

if ( crop_height_flag==ascepm_crop ) {
  // ASCE tall crop, see:
  // http://www.kimberly.uidaho.edu/water/asceewri/ASCE_Standardized_Ref_ET_Eqn_Phoenix2000.pdf
  result = 0.408 * BigDelta * (RadiationNet); // assuming ground heat flux is very small
  result = result + Gamma * 1600.0 / ( Tmean + 273.0) * WSpeed2 * vpd;
  result = result / (BigDelta + Gamma * ( 1.0 + 0.38 * WSpeed2));
}else{
  // FAO56 short crop
  result = 0.408 * BigDelta * (RadiationNet); // assuming ground heat flux is very small
  result = result + Gamma * 900.0 / ( Tmean + 273.0) * WSpeed2 * vpd;
  result = result / (BigDelta + Gamma * ( 1.0 + 0.34 * WSpeed2));
}
return (double)result;
}

void Mortons(const long Lake, const double Tmax, const double Tmin, const double SolarR, const double VaporP,
             const double Height, const double latitude, const long MyDate, const long DOY, const double PrecipAnnual,
             double *Mpotential_result, double *Mwet_result, double *Mactual_result, double *Mlake_result ){

  // Unless otherwise noted equation numbers are fromn Morton "III Modus Operandi" pages 100-143
  // See: https://www.agric.gov.ab.ca/acis/docs/mortons/mortons-evaporation-report.pdf

  // Notes:
  //       Code uses two logical variables: debug and month
  //       1 debug
  //       When debug=1, debug mode is enabled and many prints are for standard output.
  //       2 month
  //       When month=0, SolarDeclination, eta and GE are calculated using FAO56 methods.
  //       When month=1, month is be used for 2 calculations
  //           (SolarDeclination, eta) and extra-terrestrial radiation (GE)
  //           can be calculated using the exact method in Morton 1983. Currently
  //           the three are disabled.
  //       Search month for the three. Both methods generate very similar result
  //       (about or less 0.2 mm/day difference in Morton three evaporations)
  //       3 when using lake=1, Mwet is the shallow lake evaporation

```

```

//          Mpotential is point potential evaporation over water, but in a land environment
//          4 when using lake=0, Mpotential is point potential evapotranspiration.
//          Mwet is the wet-environmental areal evapotranspiration.
//          Mactual is the areal actual evapotranspiration.
// Parameters:
//      *   long      Lake,      logical variable, 0 for over land, 1 for over shallow lakes
//      *   double    Tmax,      daily maximum temperature, in degree of Celcius
//      *   double    Tmin,      daily minimum temperature, in degree of Celcius
//      *   double    SolarR,    solar radiation, in MJ per square-meter per day; MJ is Mega Joul
//      *   double    VaporP,    average vapor pressure at air temperature, in hundred Pasacal (mb)
//      *   double    Height,    station height, in meter
//      *   double    latitude,  in degrees
//      *   long      MyDate,    eight-digit date as interger like yyyymmdd
//      *   double    PrecipAnnual, annual precipitation, a rough measurement of dryness of a place,
//                          in millimeters
// On return, the following are set:
//      *   double    Mpotential, Mwet, Mactual
//          (1) When lake = 0, they are Morton Potential Evapotranspiration,
//              Morton Wet-environment areal evapotranspiration, and Morton
//              areal actual evapotranspiration, in units of mm/day
//          (2) When lake = 1, Mpotential and Mactual are not used, Mwet is the shallow lake evaporation

// local variables
double Mpotential, Mwet, Mactual;
double Tmean, SolarR_watt, LatR;
double PRatio; // PStation/PSea, the ratio is calculated using station height
double Gamma, BigDelta, VaporP_S;
long month;
double SolarDeclination, LongWaveR_Net, LongLimt, RadiationNet;
double cosZ, sinZ, cosOmega, Omega, sinOmega, cosz_small, eta, GE;
double WaterPrecip, C1, TurbCoeff, Tau, Tau_a, Tau_a_limit, G0,S_ratio, C2, Rho;
double Albedo_zd, Albedo_zz, Albedo_zz_limit, Albedo_z, Albedo_O, Albedo;

double VaporP_diff, c0_VaporP, Rtc;
double stabfac, VaporP_tc, Heat_tc;
double XTp, XVp, XBigDeltaP, Tp, Vp, BigDeltaP, Delta_Tp;
double RadiationNet_P, fz;

```

```

double temp, LatentHeat, epsigama, b0;
double VaporP_mb;

// Constants
const double Pai =3.14159;

long debug = -99;
long use_month = -99; // if use_month == 1, Morton original calculation using month is used

// Initialise output variables
*Mpotential_result = (double) -99.9;
*Mactual_result = (double) -99.9;
*Mlake_result = (double) -99.9;
*Mwet_result = (double) -99.9;

VaporP_mb = VaporP;

// ----- unit conversion
SolarR_watt = SolarR/0.0864; // Convert from MJ.m-2.day-1 to watt.m-2
LatR = latitude*Pai/180.0;

// ----- C0 assemble latitude, Height, PrecipAnnual -----
// As input arguments
// ----- C0 compute: daily mean temperature Tmean
Tmean = (Tmax+Tmin)/2.0;

// ----- C1 Compute the pressure ratio based on height:
// ----- slightly different than in Morton1
// ----- C1 Compute Psychrometric Constant (In Morton 1983)
// ----- Gamma = 0.66/1013, defined differently without Pressure
PRatio = pow( ((288.0-0.0065*Height)/288.0), 5.256); // p107 eq. 3
Gamma = 0.66*PRatio; // unit mbar.Kelvin-1
if (Tmean < 0.0)
    Gamma = 0.66/1.15*PRatio;

// ----- C2 Estimate Zenith value of the dry-season snow-free
// ----- clear-sky albedo (p107 eq. 4)

```

```

Albedo_zd = 0.26 -0.00012*PrecipAnnual*sqrt(PRatio) * (1.0+fabs(latitude/42.0)+pow( (latitude/42.0),2) );
if ( Albedo_zd < 0.11 )
    Albedo_zd= 0.11;      // most of Earth
if ( Albedo_zd > 0.17 )
    Albedo_zd = 0.17;    // deserts

// ----- C3-C5 Estimate Actual Vapor Pressure (VaporP_mb=vd),
// ----- and Saturation Vapor Pressure (VaporP_S=v) and BigDelta
// VaporP_mb and VaporP_S use unit mbar
// VaporP_S = Es_mbar(Tmax)/4 + Es_mbar(Tmin)/4 + Es_mbar(Tmean)/2
// by Francis Chiew.

VaporP_S = Es_mbar(Tmean); // (p109 eq. 5)

if ( VaporP_S < VaporP_mb ) {
    VaporP_mb = VaporP_S;
}

BigDelta = BigDeltaFunction(Tmean); // Morton 1983

// ----- C6-C11 Compute various solar angles and leading up to an estimate of
// the extra-atmospheric global radiation in W m-2
// ----- all angles in radians (Morton 1983 use degree)

SolarDeclination = 0.409*sin(2.0*Pai*(double)DOY/365.0-1.39); // Theta for Day of Year. from FAO56 p46 eq 24
if (use_month==1) {
    month = MyDate / 100 % 100;
    SolarDeclination = 23.2/180*Pai*sin((29.5*month-94.0)/180.0*Pai); // (p109, eq. 8)
}

cosZ = cos(LatR-SolarDeclination); // (p110 eq. 9)
sinZ = sin(LatR-SolarDeclination);
if (cosZ < 0.001) // (p110 eq. 9a)
    cosZ = 0.001;

cosOmega = -tan(LatR) * tan(SolarDeclination);
Omega = acos(cosOmega);

```

```

// Omega is the angle between sunrise and noon (Morton 1983),
// Omega should be same as the sunset hour angle in FAO 6, eq. 25
sinOmega = sqrt(1.0 - cosOmega * cosOmega);
if (cosOmega < -1.0)
    cosOmega = -1.0;
cosz_small = cosZ + (sinOmega/Omega-1.0)*cos(LatR)*cos(SolarDeclination); // (p110 eq. 11)

eta = 1.0 + 0.033 *cos(2.0*Pai/365.0*DOY); // Fao56
if (use_month==1)
    eta = 1.0 + 1.0/60.0*sin((29.5*month-106.0)*Pai/180.0); // (p110, eq. 12)

// GE extra-atmospheric (extra-terrestrial) global radiation (daily average).
// GE from Morton 1983 is slightly different from FAO 56.
// For the example in FAO56 p73, Morton gives 45 MJ. m-2.day-1,
// while FAO56 gives 41.09 MJ. m-2.day-1
// GE, FAO56 eq.21,P46
GE = 24.0 * 60.0 / Pai * 0.0820 * eta *
    ( Omega * sin(LatR) * sin(SolarDeclination) + cos(LatR) * cos(SolarDeclination) * sinOmega )
    / 0.0864;
//NB 0.0864 is for converting MJ.m-2.day-1 to W.m-2
//(see notes above equation 43 in III Modus opeandi)

if (use_month==1)
    GE = 1354.0 / eta / eta * Omega / Pai * cosz_small; // (p110, eq. 13)

// ----- C12 - C15 Estimate the zenith value of
// ----- snow-free clear-sky albedo(Albedo_zz),
// ----- the zenith value of clear-sky albedo (Albedo_z), and
// ----- the clear-sky albedo (Albedo)

Albedo_zz = Albedo_zd; // (p111, eq. 14)
Albedo_zz_limit = 0.5*(0.91-VaporP_mb/VaporP_S); // (p111, eq. 14a)
if (Albedo_zz > Albedo_zz_limit)
    Albedo_zz = Albedo_zz_limit;
if (Albedo_zz < 0.11 )

```

```

    Albedo_zz = 0.11;
if (Lake==1)
    Albedo_zz = 0.05;

c0_VaporP = VaporP_S - VaporP_mb; // (p111, eq. 15)
if (c0_VaporP < 0) // (15a)
    c0_VaporP = 0;
if (c0_VaporP > 1 )
    c0_VaporP = 1;

Albedo_z = Albedo_zz + (1.0 - (c0_VaporP*c0_VaporP))*(0.34 - Albedo_zz); // (p111, eq. 16)
Albedo_O = exp(1.08)-(2.16*cosZ/Pai + sinZ)*exp(0.012*(LatR-SolarDeclination)*180.0/Pai); // (p111, eq. 17.i)
Albedo_O = Albedo_O*Albedo_z;
Albedo_O = Albedo_O/1.473/(1.0-sinZ); // (p111, eq. 17.iii)

// ----- C16 - C18 estimate precipitable water vapor in millimeters and a turbid coefficient

WaterPrecip = VaporP_mb/(0.49 + Tmean/129.0); // p112 (18)

C1 = 21.0 - Tmean; // p112 (19)
if (C1 < 0)
    C1 = 0;
if (C1 > 5)
    C1 = 5;
TurbCoeff = (0.5+2.5*(cosz_small*cosz_small))*exp(C1*(PRatio-1.0)); // p112 (20)

// ----- C19 - C20 calculate the transmittancy (Tau ) and
// ----- absorption(Tau_a) of clear skies to direct solar radiation
Tau = exp (-0.089 * pow( (PRatio/cosz_small), 0.75)
    - 0.083 * pow( (TurbCoeff/cosz_small), 0.9)
    - 0.029 * pow( (WaterPrecip/cosz_small),0.6) ); // p112 (21)
Tau_a = exp( -0.0415 * pow( (TurbCoeff/cosz_small), 0.90)
    - ( pow( 0.0029, 0.5) ) * pow( (WaterPrecip/cosz_small), 0.3) ); // (22)
Tau_a_limit = exp( -0.0415 * pow( (TurbCoeff/cosz_small), 0.90)
    -0.029 * pow( (WaterPrecip/cosz_small), 0.6) ); // (22a)
if (Tau_a < Tau_a_limit)
    Tau_a = Tau_a_limit;

```

```

// ----- C21 - C22 compute the clear-sky global radiation (G0), and then the
// incident global radiation (G=SolarR_watt is an input argument
// with different unit though, no need for calculation)

G0 = GE*Tau*(1.0+(1.0-Tau/Tau_a)*(1.0+Albedo_O*Tau)); // p113 (23)

if(G0 < 0.47 * SolarR_watt) {
    return;
}

if (G0 < SolarR_watt)
    G0 = SolarR_watt;

S_ratio = 0.53*SolarR_watt/(G0-0.47*SolarR_watt); // (pdf p124, eq.43 replaces eq 24 as radiation is available)
if (S_ratio < 0.0) { // (43a)
    return;
}
if (S_ratio > 1.0) // (43a)
    S_ratio = 1.0;

// ----- C23 Estimate the average albedo
Albedo = Albedo_O*(S_ratio+(1.0-S_ratio)*(1.0-(LatR-SolarDeclination)*180.0/Pai/330.0)); // (25)

// ----- C24 - C25 Estimate the longwve solar radiation increase
// ----- due to clouds (Rho)
C2 = 10.0*(VaporP_mb/VaporP_S - S_ratio -0.42); // (26)
if (C2 < 0) // (26a)
    C2 = 0;
if (C2 > 1)
    C2 = 1;
Rho = 0.18 * ( (1.0-C2)*pow( (1-S_ratio),2.0) + C2*sqrt(1.0-S_ratio) )/PRatio; // (27)

// ----- C26 Calculate net longwave radiation at the surface
// in Morton original paper p15 near formula (28a)
// Stefan-Boltzmann constant (5.670367(13)×10-8 W m-2 K-4) x land surface emissivity of 0.92)

```

```

epsigama = 5.22e-8; // (p116, eq. 28 notes, as above)
if (Lake==1)
    epsigama = 5.5e-8;

LongWaveR_Net = epsigama* pow( (Tmean+273.0), 4.0) * (1.0 -(0.71+0.007*VaporP_mb*PRatio) *(1.0+Rho) );
LongLimt = 0.05 * epsigama*pow( (Tmean+273.0), 4.0);
if (LongWaveR_Net < LongLimt)
    LongWaveR_Net = LongLimt;

// ----- C27 Estimate net radiation

RadiationNet = (1.0-Albedo)*SolarR_watt - LongWaveR_Net;

// Estimate stability factor, and vapor transfer
// coefficient and heat transfer coefficient
// stability factor(stabfac), vapour pressure transfer
// coefficient(fa=VaporP_tc)and heat transfer coeffieient(lamda=Heat_tc)

Rtc = RadiationNet; //!unit watt.m-2
if (Rtc < 0) // (30a)
    Rtc = 0;

fz = 28.0; // unit W.m-2.mbar-1
if (Lake==1)
    fz = 25.0;
if (Tmean < 0)
    fz = fz*1.15;

VaporP_diff = VaporP_S-VaporP_mb;
if (VaporP_diff <= 0.0)
    VaporP_diff = 0.0001;
b0 = 1.00;
if (Lake==1)
    b0 = 1.12;
temp = Gamma* pow( (1.0/PRatio), 0.5) *b0*fz*VaporP_diff;
stabfac = 0.28*(1.0+VaporP_mb/VaporP_S)+BigDelta*Rtc/temp;
stabfac = 1.0/stabfac;

```

```

if (stabfac < 1.0) // (31a)
    stabfac = 1.0;

VaporP_tc = pow( (1.0/PRatio), 0.5) *fz/stabfac; // unit same as fz above
Heat_tc = Gamma + (4.0*epsigama * pow((Tmean+273.0),3.0))/VaporP_tc; // unit same as Gamma: mbar . Kelvin -1
// ----- C32-C35 loop to get Tp-----
// --- C4 loop to get Mpotential temperature
XTp = Tmean;
XVp = VaporP_S;
XBigDeltaP = BigDelta;
// converge to potential evaporation equilibrium
long ii = 1;
while ( ii<= 10){
    Delta_Tp = (RadiationNet/VaporP_tc + VaporP_mb - XVp
                + Heat_tc * (Tmean - XTp)) / (XBigDeltaP + Heat_tc);
    Tp = XTp + Delta_Tp;
    Vp = Es_mbar(Tp);
    BigDeltaP = BigDeltaFunction(Tp);
    if (fabs(Delta_Tp) < 0.00001) {
        ii=99;
    }else{
        XTp = Tp;
        XVp = Vp;
        XBigDeltaP = BigDeltaP;
    }
    ii++;
}

// ----- C5: Calculate Morton Potential Evapotranspiration -----

Mpotential = RadiationNet - Heat_tc * VaporP_tc*(Tp - Tmean); // unit Watt.m-2

// ----- C6: Calculate Morton Wet_environment Evapotranspiration ---
RadiationNet_P = Mpotential + Gamma*VaporP_tc*(Tp - Tmean);
Mwet = 14.0 + 1.2/(1.0+ Gamma/BigDeltaP)*RadiationNet_P;
// the first should be much smaller than the second term during seasons of high radiation.
if (Lake==1)

```

```

    Mwet = 13.0 + 1.12/(1.0+ Gamma/BigDeltaP)*RadiationNet_P;

// Apply constraint (40a)
if (Mwet > Mpotential)
    Mpotential = Mwet;
if (Mwet < 0.5*Mpotential)
    Mpotential = 2.0* Mwet;

// 1MJ.m-2.day-1=1.0e6/24/3600=11.574 watt.m-2
// ----- C7: Calculate Morton areal actual Evapotranspiration ---
Mactual = 2.0*Mwet - Mpotential;    // (41)
if (Lake==1)
    Mactual = 0;

LatentHeat = 28.5;    // LatentHeat of evaporation    (W-days per kilogram)
if (Tmean < 0)
    LatentHeat = 28.5*1.15;    // LatentHeat of sublimation

*Mpotential_result = (double) Mpotential/LatentHeat;

*Mwet_result = (double) Mwet/LatentHeat;
*Mactual_result = (double) Mactual/LatentHeat;

if (Lake==1)
    *Mlake_result = (double) Mwet/LatentHeat;

return;
}

```